Consumption and housing wealth breakdown of the effect of a rise in interest rates

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In this article the effect on consumption of a fall in housing wealth and housing prices, resulting from an increase in interest rates is estimated. With the help of a dynamic multiequation, macroeconomic model, the consumer response function is broken down into two parts: a direct response related to a rise in the cost of credit and another indirect one related to the deterioration of the property market. Estimation of the theoretical model is done by means of a Vectorial Error-Correction (VEC) model.

Keywords: vectorial error-correction model; wealth effect; housing prices; monetary policy

JEL Classification: C32; C51; E21

I. Introduction

In the literature two theoretical approaches have been used to analyse the role of housing in the economy: (1) life cycle models and (2) financial accelerator models.

In the ‘life cycle-permanent income’ models (Friedman, 1957; Ando and Modigliani, 1963),\textsuperscript{1} the individuals increase or decrease their wealth to maintain constant consumption. A windfall increase in wealth, whether residential or financial, will produce a permanent rise in consumption.

The financial accelerator models (Aoki et al., 2004; Iacoviello, 2005) are general equilibrium models in which housing is used as collateral for credits requested by families. An increase in their price leads to an increase in the collateral, and therefore an increase in credit and activity.

The two theoretical approaches differ in several aspects. (1) In the life cycle models, income and wealth are used as explanatory variables in the consumption equation, whereas interest rates are excluded. In the financial accelerator models, consumption depends mainly upon assets and their performance in an explicit way. Thus, interest rates and wealth play an important role in its behaviour. (2) In life cycle models, an increase in wealth has an effect on consumption, regardless of whether the effect is caused by an increase in the amount of wealth or the price. In the financial accelerator models it is the price of houses which has effects on consumption. (3) In life cycle models, the channel

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\textsuperscript{1}A theoretical deduction of the life cycle model in a context of general equilibrium of overlapping generations can be found in Gali (1990).
through which wealth affects consumption is not specified and it is implicitly assumed that agents are not restricted by either liquidity or credit. However, financial accelerator models assume that credit restrictions exist. Therefore, the way in which housing affects consumption is via the credit channel.

From an empirical viewpoint, works can be classified on the basis of the theoretical approach used. Thus, by using the life cycle theory, an equation has been specified in the form $C_t = f(Y_t, W_t)$ where $C_t$ is consumption, $Y_t$ is income and $W_t$ is wealth. By means of this equation, the effect exerted on consumption by an increase of wealth is estimated. Although the life cycle theory forecasts that the effect of wealth must be the same for every type of wealth, in practice, this equation has been estimated for different types of wealth.

Works which have estimated marginal propensity to consume housing wealth can be classified among those estimating a long-term relationship, such as Case et al. (2005) or Ludwing and Slok (2004) and those who analyse the short-term adjustment by using an Error-Correction Model (ECM) such as Barata and Pacheco (2003) and Catte et al. (2004). Though there are very diverse findings, the conclusion can be drawn that, in most of the works there exists a significant effect of housing wealth on consumption.

Most of the empirical works based on the financial accelerator model use the Vector Autoregressive (VAR) methodology for studying the relationship between consumption, housing and interest rates.

Lastrapes (2002) estimates a VAR model for the American economy in which a fall in interest rates gives rise to an increase in house prices and housing investment.

Aoki et al. (2002) find that for the English economy, an interest rate rise produces a drop in housing prices, residential investment and consumption of durable and nondurable goods.

Finally, Iacoviello (2005) estimates a VAR model for the American economy in which a negative Gross Domestic Product (GDP) and house price response is found in the face of an increase in interest rates. Moreover, a positive shock in housing prices causes rise in GDP.

There are two important differences between both empirical approaches. (1) In works based on the life cycle theory no specification is made of the reason for wealth variation, whereas in works based on the financial accelerator model the wealth varies in the face of an interest rate shock. (2) In the case of life cycle models, the relevant variable is housing wealth. This variable is a measurement of both the number of dwellings and their price. In the case of financial accelerator models, the relevant variable is some measurement of housing prices.

Finally, there exists a group of empirical works which do not specify a particular theoretical model and are concerned with the statistical properties of variables themselves. These works, which share the ideas of the life cycle models, use a log-linear approach of agents’ budgetary restriction to show that there exists a cointegration relationship between consumption, wealth and income. Subsequently, by using the previous cointegration relationship, a Vectorial Error-Correction (VEC) model is specified. The prediction error variance is broken down in this model into a permanent and transitory factors. Two important results are observed: (1) when there are imbalances in the cointegration relationship, the variable adjusted is wealth and (2) consumption is explained by permanent factors, so only permanent changes in wealth affect consumption.

Picchette and Tremblay (2003) analyse the Canadian case making a distinction between housing and financial wealth. This separation makes possible the conclusion that housing wealth is more important for explaining consumption since the prediction error variance of this wealth is made up of permanent factors. Fernandez-Corugedo et al. (2003) analyse the English case by separating durable and nondurable goods. Lettau and Ludvigson (2004) analyse the case of the American economy, Hamburg et al. (2008) analyse the German case and Chen (2006) analyse the Swedish one. Whatever the approach used, wealth seems to be an important explanatory variable for consumption.

In this article we follow the financial accelerator approach, where the variations in wealth come from a known source, a shock in the interest rate. As empirical findings and theoretical models show, a fall in interest rates might lead to a rise in house price, housing investment and finally to a rise in consumption of durable and nondurable goods. Also, as pointed out in Iacoviello (2005), a fall in interest rate may lead to a rise in inflation that, by reducing real debt commitments, it may lead to a rise in consumption.

All these positive effects on consumption are not independent of each other, i.e. the final effect of a fall in interest rates has many components. A direct effect associated to a cheaper credit and some indirect effects associated to the rise in housing investment, house prices and inflation, as well as the likely feedback reaction of the central bank, omitted from the Iacoviello paper. Our objective is to evaluate the relative importance of all these variables in the final response of consumption to a rise in interest rates.
In this article the role of housing wealth in the Spanish economy is analysed. For that purpose, a theoretical macroeconomic framework (adapted from that used in Flores et al. (1998)) is proposed which, by using the relevant variables of the financial accelerator model, enables the response functions of consumption, housing wealth and the price of housing when faced with a permanent, unit increase in the level of interest rates to be estimated.

Moreover, with the help of this dynamic theoretical framework, the final reaction of consumption is identified and broken down into two parts: (1) the direct reaction, related to the increase in the cost of credit and (2) the indirect reaction related to variations in housing wealth and the fall in the price of houses. Finally, the above-mentioned framework parameters and reactions are estimated from a VEC model. This breakdown is the fundamental contribution of this article.

Unlike the works on life cycle, in this article, housing wealth is considered as an endogenous variable, which may have feedback relationships with the remaining variables in the agent’s set of information. Moreover, the model presented here enables the total effect of housing work to be divided into two parts: (1) an effect brought about by the increase in the stock of new housing and (2) an effect brought about by the increase in its value. Regarding the financial accelerator models, our model makes it possible to analyse through which channels an increase in interest rates affect consumption whilst making it possible to evaluate the importance of both the price of housing and the stock of housing on consumption. In the empirical analysis special attention has been devoted to the statistical properties of the data, that is, orders of integration, existence of cointegration relationships, effects of possible extreme values, presence of feedback, etc.

All these properties, with no need to restrict any of them, have been incorporated into the theoretical framework which admits a VAR-type stochastic multivariate representation. This representation can be estimated consistently from the corresponding empirical model. To identify the parameters of the theoretical framework, it is only necessary to assume that there is an asymmetry in the information used by the different agents when determining the values corresponding to them. That is, private sector agents decide consumption levels, housing wealth and prices in each period, being aware of the level of interest rates, whereas the central bank determines interest rates without knowing the exact values which the variables determined by private agents will take.

Once the reaction of the variables has been estimated, it is concluded that a permanent 1% rise in the interest rate produces a permanent fall in consumption, in the growth of housing wealth and housing inflation. Moreover, when the components of consumer response are analysed, it is seen that the final long-run effect of the interest rate on consumption (a 1.33% fall) can be broken down into a 0.53% fall due to the direct effect occasioned by the increased cost of credit, a 0.96% fall due to the drop in housing wealth, a 0.48% fall as a result of the fall in housing prices and a 0.66% rise due to the feedback reaction of the central bank that lowers rates when inflation falls. The expansion of the consumption and the real state market, experienced by the Spanish economy since mid-1990s, could be well explained by the continuous fall of interest rates (along with the low level of indebtedness of Spanish families).

This article is organized in the following manner. In Section II, the theoretical framework is specified with which reactions of variables to interest rate shocks can be identified. In Section III, data and previous statistical findings are presented. In Section IV the VEC model is constructed. In Section V, there are estimates of the response of variables to a permanent unit rise in interest rates. In Section VI, the different components of the final consumer reaction to an interest rate increase are estimated, and in Section VII, the main conclusions of this article are presented.

II. Theoretical Framework

In this section, the theoretical model put forward in Flores et al. (1998) and Pereira and Flores (1999) is adapted for the aim of this article.

Two types of agents are considered in this economy, private sector and central bank agents.

It is assumed that private agents in each period determine the consumption levels \( C_t \), housing wealth \( W_t \) and price of housing \( P_{W_t} \).

With the aim of facilitating the theoretical model–empirical model integration, we shall assume that private agents determine the vector \( z_t = (c_t, \ln W_t, \ln P_{W_t}) \), where the small letter variables denote the logarithm of the levels of the corresponding capital letter.

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2 In fact, it is not necessary to assume that agents choose \( \ln W_t \) or \( \ln P_{W_t} \), instead of \( w_t \) or \( p_{P_{W_t}} \). The reason for maintaining this assumption is our wish to work with a vector \( z_t \) of integrated variables of order 1, \( I(1) \), since both \( w_t \) and \( p_{P_{W_t}} \) are \( I(2) \).
variables, with $\nabla = 1 - B$ where $B$ is the rational lag operator.

Moreover, it is assumed that the central bank determines the interest rate levels ($r_t$) in each period. Both private agents and the central bank are aware, at the beginning of the period, of past values for all the variables. Nonetheless, private agents fix $c_t$, $\nabla w_t$, and $\nabla p_{vt}$, when aware of the interest level for the period, whereas the central bank determines the interest rate without knowing the values of $c_t$, $\nabla w_t$, and $\nabla p_{vt}$.

This assumption is crucial to be able to identify the relevant parameters in the theoretical model. The assumption does not seem to be excessively restrictive given the information provided by central banks to the rest of the economic agents nowadays.

The choice of the relevant variables making up the agents’ set of information data in this model is similar to that made in typical financial accelerator models.

**Representation of private agents’ behaviour in mathematical form**

The set of information available to private agents ($\Omega_{zt}$) is made up of past values of $z_t$ as well as past and present values of $r_t$, that is,

$$\Omega_{zt} = \{z_{t-j}, r_{t-1}, r_t\}, \quad j = 1, 2, 3, \ldots$$

In each period, private agents determine the level of $c_t$, $\nabla w_t$, and $\nabla p_{vt}$ using the information of $\Omega_{zt}$. This means that $z_t$ depends on the present and past values of $r_t$ as well as the past ones of $z_t$, that is,

$$z_t = v_z(B)r_t + \varepsilon_{zt}$$

(1)

where $v_z(B) = (v_z(B), v_y(B), v_y(B))$ is a $(3 \times 1)$ vector of stable transfer functions. Each of them registers the unidirectional reaction of the corresponding variable to a transitory unit variation (impulse) in the interest rate. Each of them has the form $v_j(B) = v_{j0} + v_{j1}B + v_{j2}B^2 + \cdots$ for $j = c, w, p_v$. \(\varepsilon_{zt} = (\varepsilon_{crt}, \varepsilon_{wtr}, \varepsilon_{pvt})\) is a vector of random variables which follows a multivariate VAR-type stochastic process. $\pi_z(B) = I - \pi_1B - \pi_2B^2 - \cdots$ is a polynomial $(3 \times 3)$ matrix whose determinant of which may have roots on the unit circle, that is, the variables of the vector $z_t$ may not be stationary.

Finally, $\alpha_{zt} = (\alpha_{zt}, \alpha_{wtr}, \alpha_{pvt})$ is a white noise vector with contemporary covariance matrixes $\Sigma_z$.

**Mathematical representation of the central bank’s behaviour**

The set of information for the central bank ($\Omega_{rt}$) is made up of the past values of $r_t$ and only for the past ones of $z_t$, that is,

$$\Omega_{rt} = \{r_{t-j}, z_{t-j}\}, \quad j = 1, 2, 3, \ldots$$

In each period the central bank determines $r_t$ using the information set of $\Omega_{rt}$. This means that $r_t$ depends on the past values of $r_t$ and $z_t$,

$$r_t = v_r(B)z_t + \varepsilon_{rt}$$

(2)

where $v_r(B) = (v_r(B), v_y(B), v_y(B))$ is a $(1 \times 3)$ transfer functions vector. Moreover, $\varepsilon_{rt}$ is a random scale variable following a general Autoregressive Integrated Moving Average (ARIMA) ($p, d, q$)-type univariate process; $\pi_r(B)$ is a scale polynomial whose roots may be on the unit circle and $\alpha_{rt}$ is a scale white noise, with variance $\sigma_r^2$ and independent of the elements of $\alpha_{zt}$.

It is important to highlight that the restriction $v_r(0) = v_y(0) = v_y(0) = 0$ in Equation 2, a result of the assumption made on the set of information from the central bank as well as the assumption of independence between $\alpha_{rt}$ and $\alpha_{zt}$, constitutes sufficient restrictions to identify the parameters of the reaction functions of the variables of the vector $z_t$ to an impulse in the interest rate.

**Complete theoretical model in VAR form**

The models (1) and (2) can be written in matrix form as

$$\begin{bmatrix} \pi_z(B) & -\pi_z(B)v_z(B) \\ \pi_r(B) & \pi_r(B) \end{bmatrix} \begin{bmatrix} z_t \\ r_t \end{bmatrix} = \begin{bmatrix} \alpha_{zt} \\ \alpha_{rt} \end{bmatrix}$$

(3)

In compact notation,

$$\Pi_z(B)y_t = \alpha_{zt}$$

(4)

In the case of interest rate the small letter denotes $r_t = \ln(1 + R_t)$.

The evolution of the variables $c_t$, $\nabla w_t$, and $\nabla p_{vt}$ is interpreted as the result of the optimization problem of the theoretical financial accelerator models, in which private agents have to decide between two goods, to consume or buy housing, when the relative price of both goods is $p_{vt}$.

The reason for not making any assumption on the transfer function and on $\pi_z(B)$ is that the proposed model allows variables not to be stationary. The data will determine whether such a polynomial has unit roots or not.

It is assumed that the monetary policy of this economy is to determine the interest rate instead of money supply.
Consumption and housing wealth

where Σ is the contemporary matrix of variances and covariances of the error term.

The multivariate model proposed in (3) is not normalized in the sense of Jenkins and Alavi (1981) since

$$\Pi_s(0) = V = \begin{bmatrix} I & -v_{0a} \\ 0 & 1 \end{bmatrix}$$

(5)

where $v_{0a} = (v_{10}, v_{1y}, v_{1p})^T$ is the vector of contemporaneous effects of $r_t$ on $z_t$.

The model can be normalized by premultiplying (3) by $V^{-1}$

$$\Pi_s^*(B) y_t = \alpha^*_s y_t$$

(6)

where

$$\Pi_s^*(B) = V^{-1} \Pi_s(B)$$

and the covariance matrix of $\alpha^*_s$ is

$$\sum^* = V^{-1} \sum (V^{-1})^T = \begin{bmatrix} \sum_{zz} + v_{0a} v_{0a}^T & v_{0a} \sigma^2 \\ v_{0a}^T \sigma^2 & \sigma^2 \end{bmatrix}$$

(7)

The model (6) with covariance matrix (7) is a normalized VAR model.

Impulse response functions

Using (3), the vector $z_t$ can be written as

$$z_t = \Psi_s(B) x_{0r} + \Psi_s(B) x_{0z}$$

(8)

where

$$\Psi_s(B) = [I - v_z(B) v_z(B)]^{-1} v_z(B) \pi_s(B)^{-1}$$

$$= \Phi_{0r} + \Phi_{1r} B + \Phi_{2r} B^2 + \ldots$$

(9)

$$\Psi_z(B) = [I - v_z(B) v_z(B)]^{-1} \pi_s(B)^{-1}$$

$$= I + \Phi_{1z} B + \Phi_{2z} B^2 + \ldots$$

(10)

The matrix $\Psi_s(B)$ is a polynomial 3 x 1 matrix

$$\Psi_s(B) = \begin{bmatrix} \Psi_{nr}(B) \\ \Psi_{nr}(B) \\ \Psi_{npr}(B) \end{bmatrix}$$

(11)

The sequence of coefficients associated with the polynomials $\Psi_s(B)$ of Equation 9 are interpreted as the reaction function of the variables of vector $z_t$ to an impulse in $z_{0z}$, that is $\partial z_t / \partial z_{0z,j}$ for $j = 0, 1, 2, \ldots$. This function measures the effects of a shock in $r_t$ on the variables of $z_t$. Estimating this function is the key for describing the effects of interest rate on consumption, house prices and housing wealth.

It is important to note that in order to analyse the effects of interest rates, it is not necessary to specify a whole structural model; all that is needed is models represented by (3) and (4). Nonetheless, if the intention is to calculate the effect that a shock in any of the variables determined by the private sector may have on the rest of the variables, it will be necessary to make extra assumptions, tending towards the complete diagonalization of the matrix $\Sigma$.

Breakdown of consumption reaction

By developing in detail the expression of the model (3), it can be observed how this system takes the following form:

$$\begin{bmatrix} \pi_{11}(B) & \pi_{12}(B) & \pi_{13}(B) & \pi_{14}(B) \\ -\pi_{21}(B) & \pi_{22}(B) & \pi_{23}(B) & \pi_{24}(B) \\ -\pi_{31}(B) & -\pi_{32}(B) & \pi_{33}(B) & \pi_{34}(B) \\ -\pi_{41}(B) & -\pi_{42}(B) & -\pi_{43}(B) & \pi_{44}(B) \end{bmatrix} * \begin{bmatrix} c_t \\ v_{0r} \\ v_{0p} \\ r_t \end{bmatrix} = \begin{bmatrix} \alpha_{c+} \\ \alpha_{v+} \\ \alpha_{p+} \\ \alpha_{r+} \end{bmatrix}$$

(12)

with

$$\pi_{14}(B) = -\pi_{11}(B) \pi_{14}(B) + \pi_{12}(B) \pi_{14}(B) + \pi_{13}(B) \pi_{14}(B)$$

$$\pi_{24}(B) = -\pi_{21}(B) \pi_{24}(B) + \pi_{22}(B) \pi_{24}(B) + \pi_{23}(B) \pi_{24}(B)$$

$$\pi_{34}(B) = -\pi_{31}(B) \pi_{34}(B) + \pi_{32}(B) \pi_{34}(B) + \pi_{33}(B) \pi_{34}(B)$$

Given Equation 12, the final response of consumption to a shock in $r_t$ can be written as

$$\Psi_{nr}(B) = \Gamma_r(B) + \Theta_v(B) + \Theta_{vp}(B) + \gamma_{v+}(B) + \gamma_{p+}(B)$$

(13)

Equation 13 shows that the final reaction of consumption, $\Psi_{nr}(B)$, can be broken down into the following: (1) a direct effect of interest rates on consumption, represented by $\Gamma_r(B)$; (2) an indirect effect coming from variations in $W$ and $PV$, given by $\Theta_v(B) + \Theta_{vp}(B)$ and (3) a feedback effect, given by $\gamma_{v+}(B) + \gamma_{p+}(B)$, due to the reaction of the central bank.

The indirect effect has two components:

(2.1) The unidirectional effect through the residential wealth, defined as

$$\Theta_v(B) = -\frac{\pi_{12}(B)}{\pi_{11}(B)} v_{0r}(B)$$
\( \Theta_d(B) \) arises as the combination of two effects: the effect of \( r_t \) on the residential wealth, represented by \( v_{rt}(B) \), and the effect of the residential wealth on consumption represented by \( \pi_{w}(B) \) and

\[
\Theta_p(B) = -\frac{\pi_{w}(B)}{\pi_{r}(B)} v_{pv}(B)
\]

(2.2) The unidirectional effect through \( pv_t \) (price of housing or collateral effect), defined as

\[
\Theta_{pv}(B) = -\frac{\pi_{w}(B)}{\pi_{r}(B)} v_{pv}(B)
\]

\( \Theta_{pv}(B) \) also arises as the combination of two effects: the effect of \( r_t \) on \( pv_t \), represented by \( v_{pv}(B) \) and the effect of \( pv_t \) on consumption, given by \( \pi_{w}(B) \).

Also, the feedback effect has two components:

\[
\begin{align*}
\Theta_{csv}(B) &= \frac{v_{c}(B)v_{w}(B)v_{sv}(B)}{1 - [v_{w}(B)v_{c}(B) + v_{sv}(B)v_{pv}(B)]} \\
\Theta_{cpv}(B) &= \frac{v_{c}(B)v_{pv}(B)v_{pv}(B)}{1 - [v_{pv}(B)v_{c}(B) + v_{pv}(B)v_{pv}(B)]}
\end{align*}
\]

**Estimation strategy**

The model proposed in (6) and (7) is the VAR model on the levels of the variables, which can be estimated directly from the data.

By matching the matrix of variances and covariances as estimated from the error term of the VAR model, to expression (7), \( V \) can be estimated. Once \( V \) has been estimated, the remaining parameters of the model proposed in (3) and (4) can be uniquely estimated from the estimates of (6). Finally, the impulse response function and its components can be estimated using (9) and (13), respectively.

### III. Empirical Analysis

**Data**

Annual data on the Spanish economy, for the period 1974 to 2002,\(^8\) has been used. The variables are:

- **C**: ‘Domestic consumption’. This series is built-up by the Instituto Nacional de Estadística (INE), according to the Sistema Europeo de Cuentas 95 (SEC-95) methodology which standardizes the annual accounts of EU countries. The data are obtained from the Economics ministry on the website http://www.meh.es/es-ES/Estadistica%20e%20Informes/Paginas/estadisticasV2.aspx.

- **R**: ‘MIBOR 1-Month’, obtained from the Bank of Spain (http://www.bde.es/webbde/es/estadis/estadis.html).\(^9\)

- **W**: ‘Stock of real net housing wealth’ (millions of 1995 euros). These data are estimates made by the Instituto Valenciano de Investigaciones Económicas (IVIE) and Banco Bilbao Vizcaya (BBVA) (Mas et al., 2007) (http://www.ivie.es/banco/stock.php and http://www.fbbva.es/TLFU/tlfu/areas/econosoc/bbdi/capital.jsp).

- **PV**: ‘Implicit deflator of housing wealth’, measured as the ratio between nominal and real stock. Both series are obtained from IVIE and BBVA.

**Univariate analysis and integration orders**

In Figs (1)–(4) the series \( c_t, w_t, p_{rt}, \) and \( r_t \) are presented. The series are clearly nonstationary. Table 1 shows the Augmented Dickey–Fuller (ADF) test for the first differences of these variables.

The findings suggest that the \( V_{ct} \) and \( V_{rt} \) series are stationary, \( r(0), \) since the value of the statistic, \(-3.08\) and \(-4.32, \) is less than the critical value at 95%
The \( r_{pvt} \) series is clearly nonstationary since the value of the statistic for any number of lags is lower than the critical value. The \( r_{wt} \) series is nonstationary since its statistic for \( p = 3 \) is –2.27, less than the critical value of the tables. However, since with other values of \( p \) the result is ambiguous, the graphs of \( r_{pvt} \) and \( r_{wt} \) are presented in Figs 5 and 6.

In both figures it can be seen that the variables \( r_{pvt} \) and \( r_{wt} \) are nonstationary since they show a negative trend.

In Table 2, the estimates of the univariate Autoregressive Moving Average (ARMA) models for the stationary series are presented.

### Table 1. ADF test for the V series

<table>
<thead>
<tr>
<th>ADF</th>
<th>( p = 1 )</th>
<th>( p = 2 )</th>
<th>( p = 3 )</th>
<th>( p = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{ct} )</td>
<td>-3.08</td>
<td>-2.72</td>
<td>-2.58</td>
<td>-4.39</td>
</tr>
<tr>
<td>( V_{w} )</td>
<td>-4.54</td>
<td>-4.59</td>
<td>-2.27</td>
<td>-3.09</td>
</tr>
<tr>
<td>( V_{pr} )</td>
<td>-1.50</td>
<td>-1.98</td>
<td>-1.61</td>
<td>-2.01</td>
</tr>
<tr>
<td>( V_{r} )</td>
<td>-4.32</td>
<td>-3.68</td>
<td>-3.47</td>
<td>-2.02</td>
</tr>
</tbody>
</table>

**Notes:**
- \( H_0: \rho = 1 \) in the model \( \nabla^p z_t = \mu + \rho \nabla z_{t-1} + \sum_{j=1}^{p} \gamma_j \nabla z_{t-j} + u_t \). The critical value at 95% is \( -2.96 \) (MacKinnon). For \( r_{pvt}, \mu = 0 \) and the critical value at 95% is \( -1.95 \) (MacKinnon).

The \( V_{pr} \) series is clearly nonstationary since the value of the statistic for any number of lags is lower than the critical value. The \( V_{wt} \) series is nonstationary since its statistic for \( p = 3 \) is –2.27, less than the critical value of the tables. However, since with other values of \( p \) the result is ambiguous, the graphs of \( V_{pr} \) and \( V_{wt} \) are presented in Figs 5 and 6. In both figures it can be seen that the variables \( V_{pr} \) and \( V_{wt} \) are nonstationary since they show a negative trend.

In Table 2, the estimates of the univariate Autoregressive Moving Average (ARMA) models for the stationary series are presented.
It is important to stress the lack of Moving Average (MA) operators in the univariate models for $r_{wt}$ and $r_{pvt}$. The appearance of MA terms close to invertibility would indicate a possible over-differentiation problems. The ADF test and the univariate models indicate that the series $ct$ and $rt$ are integrated of order 1, $I(1)$. The ADF test, the graphic analysis and the estimation of the univariate models show sufficient evidence in favour of the series $wt$ and $pvt$ being $I(2)$.

### IV. Empirical VAR Model

#### Choice of VAR order

In Table 3 the statistic of the Likelihood Ratio (LR) for the different orders of VAR is shown. Due to the fact that the value of the statistic for order 3 is 26.02 higher than the critical value at 90% confidence, the null hypothesis that all coefficients of VAR(3) are zero can be rejected. Thus, the LR test suggests a VAR(3) as the most suitable model.

Additionally, the residual crossed correlation functions corresponding to VAR(2) and VAR(3) are calculated. These functions are presented in Figs 7 and 8. Some residual-crossed correlations of VAR(2) have significant values which disappear when a VAR(3) is chosen.

#### Cointegration

In this section, possible cointegration relationships are analysed. Assuming that there exists a constant both in the VAR and in the cointegration relationship, Johansen’s (1991) test showed two cointegrating equations:

\[
\nabla_{(0.02)} r_{wt} - 0.16 \nabla_{(0.02)} r_{pv} + 0.16 r_{rt} = \zeta_{(0.02)}
\]

\[
 c_{(2.03)} - 7.61 \nabla_{(2.42)} r_{pv} + 1.04 r_{rt} = \zeta_{(2.42)}
\]

The relationships $\zeta_{1t}$ and $\zeta_{2t}$ are shown in Figs 9 and 10. These show that $\zeta_{1t}$ can be considered as a...
cointegration relationship but $\zeta_2 t$ cannot, since it shows a clear positive trend.

Due to the great sensitivity of the Johansen’s test to degrees of freedom and for the purpose of making the findings more robust, the methodology of Engle and Granger (1987) is proposed to detect possible cointegration relationships. This methodology is less powerful than the Johansen’s test but less sensitive both to the choice of the order in the multivariate model and to degrees of freedom.

In Table 4, the ADF test for the residual of the regression of each nonstationary variable with the remaining ones is presented. If residuals are stationary, the regression shows a cointegration relationship.

From Table 4 it can be concluded that there exists a cointegration relationship between $\nabla w_t$ and the other variables.\(^{11}\) To analyse which variables should be included in the cointegration relationship, in Table 5 the ADF test for the residuals of the regression of $\nabla w_t$ is presented with the other variables excluded one by one.

From Table 5 it can be concluded that $c_t$ should not be in the relationship. The Ordinary Least Squares (OLS) estimation of this relationship is presented in Equation 16.

\[
\nabla w_t = 0.021 + 0.13 \nabla p_{rt} - 0.5 r_t + \xi_{1t} + \xi_{2t} \quad (16)
\]

\(^{11}\) It could also be concluded that there is a relationship of $\nabla p_{rt}$ with the other variables, but later analysis showed that it was the same relationship as with $\nabla w_t$ normalized in another way.
The cointegration relationship estimated in Equation 16 is presented in Fig. 11. This relationship is almost identical to Equation 14, obtained by the Johansen method. As both methods have the same cointegration relationship, it is concluded that this relationship exists and is the only one of its kind.

The latter is a stationary relationship between the growth rate of housing wealth, the growth rate of house prices and the interest rate. This relationship can be interpreted as the supply of housing in that economy where new housebuilding, as calculated by the growth rate of housing wealth, positively depends on the increase in house prices and negatively on the interest rate.

**VEC model**

By using the cointegration relationship shown in Equation 16, an equation-by-equation specification is given for the corresponding VEC model. The estimation takes place in two stages. First, the cointegration relationship

\[
\begin{align*}
\varepsilon_{ct} &= r_{ct} - 0.13 r_{w} - 0.05 r_{t} \\
\varepsilon_{ct-1} &= 0.47 \\
\varepsilon^2_{w} &= 0.12 \\
\varepsilon^2_{p} &= 0.38 \\
\varepsilon^2_{r} &= 0.16 \\
\varepsilon^2_{ct-2} &= 2.31 \\
\varepsilon^2_{w-2} &= 0.23 \\
\varepsilon^2_{p-2} &= 0.38 \\
\varepsilon^2_{r-2} &= 0.03
\end{align*}
\]

is estimated and, subsequently, the remaining parameters are jointly estimated (Lütkepohl, 2006). The results are given in Table 6.

The covariance matrix is presented in Equation 17 and the instant correlations matrix is presented in Equation 18. The confidence bands for the instant correlations at 95% are \(\pm 2^{*}\text{SD} = \pm 2/\sqrt{n} = \pm 0.38\).
The residuals graph is presented in Fig. 12 and the functions of residuals crossed correlations in Fig. 13. Significant correlations are not found in them, so these findings suggest that model (6) adequately represents the dynamic correlations structure among the variables.

The model in Table 6, once expressed as a nonstationary VAR on the variables of the vector $z_t$, turns out to be the estimated version of the normalized model (6). The covariance matrix shown in Equation 17 is precisely the estimated version of Equation 7. From this covariance matrix, $V$ can be estimated.

$$\rho(0) = \begin{pmatrix} 1.00 & 0.42 & 0.43 & -0.23 \\ 0.42 & 1.00 & 0.26 & -0.39 \\ 0.43 & 0.26 & 1.00 & 0.02 \\ -0.23 & -0.39 & 0.02 & 1.00 \end{pmatrix}$$ (18)

Premultiplying the VAR version of the model in Table 6 by $V$, the orthogonalized model deduced from Equation 3, along with the diagonal covariance matrix deduced in Equation 4, can be obtained. This orthogonalized model, in its VEC version, is presented in Table 7 along with the instant correlations which are presented in matrix (20).

$$\rho(0) = \begin{pmatrix} 1.00 & 0.36 & 0.44 & 0.00 \\ 0.36 & 1.00 & 0.28 & -0.02 \\ 0.44 & 0.28 & 1.00 & 0.01 \\ 0.00 & -0.02 & 0.01 & 1.00 \end{pmatrix}$$ (20)

This model shows the presence of dynamic relationships among all the variables. In Equation 1 it is observed that when a shock in interest rates takes place, the residential wealth growth rate responds immediately. Nevertheless, the growth rate of housing prices takes 2 years to react. As can be seen in Equation 3, the maladjustments in the cointegration relationship affect the growth rate of housing price rises which adapt to achieve the new equilibrium. Equation 4 of the VEC model shows that the interest rate is not strictly exogenous since it is affected by housing prices. If we assume that housing inflation may be a leading indicator of general inflation,
Equation 4 would reflect the reaction of the central bank in the face of variations in the latter. Finally, Equation 1 indicates that consumption does not contribute to the balance of the new situation, since it does not affect either housing wealth or housing prices, but it is affected by their evolution and that of interest rates.

Therefore, faced with an increase in interest rates, an imbalance occurs between $\nabla w_t$ and $\nabla p_t$. This imbalance is corrected with an immediate fall of $\nabla w_t$ (as in empirical life cycle models) and after 2 years, with a fall in $\nabla p_t$. The rise in interest rate, as well as the fall in $\nabla w_t$ and $\nabla p_t$, gives rise to a new equilibrium among the variables in the real estate sector, with consumption levels below the initial ones. This effect is eased as a consequence of the central bank’s reaction in reducing interest rates when there is a fall in $\nabla p_t$.

### V. Reaction Functions to the Interest Rate

On the basis of the orthogonalized model (3), the estimate of which is presented in Table 7, the reaction function to the normalized impulse proposed in Equation 8 is calculated. The reaction function to the step change (i.e. the reaction function to a permanent unit rise in interest rates), is presented in Table 8.

As observed in Fig. 14\textsuperscript{12} as well as in Table 8, a permanent increase in interest rates does not generate any instant effect on the growth rate of housing prices. In fact, there is a 2-year dead period until housing inflation begins to react. In the third period a 0.07 percentage point fall in the increase in housing prices occurs and from that time onwards, there is a steady drop during seven periods of as much as 0.9

\textsuperscript{12} The bands of confidence of the reactions at 95\% are sketched in by discontinuous lines. In the Appendices 1, 2 and 3 the method used to calculate is described and the findings are presented.

\begin{table}[h]
\centering
\caption{VEC orthogonalized model}
\begin{tabular}{lllll}
\hline
Dependent variable & $\nabla c_t$ & $\nabla^2 w_t$ & $\nabla^2 p_t$ & $\nabla r_t$ \\
\hline
$\mu$ & 0.015 & -0.12 & & \\
$ecm_{t-1}$ & 0.47 & & 5.84 & \\
$\nabla c_{t-1}$ & 0.19 & 0.01 & 0.38 & \\
$\nabla^2 c_{t-1}$ & -0.16 & -0.03 & & \\
$\nabla^2 w_{t-1}$ & 0.23 & & & \\
$\nabla^2 p_{t-1}$ & -0.03 & -0.02 & & \\
$\nabla^2 c_{t-2}$ & & & & \\
$\nabla^2 w_{t-2}$ & & & & \\
$\nabla^2 p_{t-2}$ & & & & \\
$\nabla r_{t-2}$ & & & & \\
\hline
\end{tabular}
\end{table}
percentage points, where it stays permanently. The fall in housing prices is steady, and after 25 years it stands at 19.27%. These findings imply that house prices are very sensitive to interest rates, thus supporting the theories pointing to financial conditions as the determining components of house prices. Thus, the rise in the price of housing since the mid-1990s could be partly accounted for by the steady fall in interest rates.

As can be seen in Fig. 15 as well as in Table 8, a permanent rise in interest rates produces an immediate fall of 0.02 percentage points in the growth rate of housing wealth. Such a rate, interpreted as new housing starts, continues to fall till it reaches 0.15 pp, which is permanent in the long term. Unlike what happened with housing inflation, where the reaction was prolonged steadily over a period of time, the reaction of housing wealth is produced over a small number of periods. Thus, in 3 years there is a 70% reaction and in 4 years nearly 90%, showing a highly rapid adjustment. The drop in housing wealth, after this reaction and in 4 years nearly 90%, showing a highly rapid adjustment. The drop in housing wealth, after

| Table 8. Reaction to a permanent interest rate shock |
|---|---|---|---|---|---|---|---|---|
| Years | $\n\delta C_t$ | $\n\delta w_t$ | $\n\delta p_{tr}$ | $\n\delta r_t$ | $\n\delta c_{mt}$ | $\n\delta c_t$ | $\n\delta w_t$ | $\n\delta p_{tr}$ | $\n\delta r_t$ |
| 1 | -0.13 | -0.02 | 0.00 | 1.00 | 0.03 | -0.13 | -0.02 | 0.00 | 1.00 |
| 2 | -0.22 | -0.06 | 0.00 | 0.00 | -0.01 | -0.13 | -0.02 | 0.00 | 1.00 |
| 3 | -0.16 | -0.11 | -0.07 | 0.00 | -0.05 | -0.13 | -0.02 | 0.00 | 1.00 |
| 4 | -0.17 | -0.14 | -0.37 | -0.03 | -0.04 | -0.13 | -0.02 | 0.00 | 1.00 |
| 5 | -0.23 | -0.16 | -0.60 | -0.11 | -0.03 | -0.13 | -0.02 | 0.00 | 1.00 |
| 6 | -0.18 | -0.16 | -0.80 | -0.09 | -0.02 | -0.13 | -0.02 | 0.00 | 1.00 |
| 7 | -0.15 | -0.17 | -0.93 | -0.08 | -0.01 | -0.13 | -0.02 | 0.00 | 1.00 |
| 8 | -0.09 | -0.16 | -0.99 | -0.05 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 9 | -0.05 | -0.16 | -1.00 | -0.02 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 10 | -0.01 | -0.15 | -0.99 | 0.00 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 11 | 0.01 | -0.15 | -0.96 | 0.01 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 12 | 0.02 | -0.15 | -0.93 | 0.01 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 13 | 0.02 | -0.15 | -0.91 | 0.01 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 14 | 0.01 | -0.15 | -0.89 | 0.01 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 15 | 0.01 | -0.15 | -0.89 | 0.01 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 16 | 0.01 | -0.15 | -0.89 | 0.00 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 17 | 0.00 | -0.15 | -0.89 | 0.00 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 18 | 0.00 | -0.15 | -0.89 | 0.00 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 19 | 0.00 | -0.15 | -0.89 | 0.00 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 20 | 0.00 | -0.15 | -0.90 | 0.00 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 21 | 0.00 | -0.15 | -0.90 | 0.00 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 22 | 0.00 | -0.15 | -0.90 | 0.00 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 23 | 0.00 | -0.15 | -0.90 | 0.00 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 24 | 0.00 | -0.15 | -0.90 | 0.00 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |
| 25 | 0.00 | -0.15 | -0.90 | 0.00 | 0.00 | -0.13 | -0.02 | 0.00 | 1.00 |

Note: *The effect in the second period on $\n\delta p_{tr}$ estimated is 0.16 but due to it being within the bands of confidence the idea of it being 0 cannot be rejected.
As can be observed in Fig. 16 as well as in Table 8, a permanent interest rate increase gives rise to a transitory fall in the consumption growth rate, since its reaction peters out over time (16 years). An increase in rate produces an immediate 0.13% fall. This fall increases over the following period and after a brief rally at 5 years the fall in the growth rate of consumption declines to 0.

Consumption, as can be seen in Fig. 17, falls immediately by 0.13%. The fall in consumption grows steadily till it reaches 1.33%, in the long term. As far as the speed of reaction is concerned, it can be seen that in 5 years 68% of the consumption reaction takes place and in 7 years 93%, showing a relatively quick adjustment. Finally, it is important to note that albeit in the long run the effect may be a 1.33% fall, the fact that the rate only increases by 0.66% means that the effect of consumption of a one point interest rise is 1.98%. Such a fall is similar to the drop in consumption in the 1993 crisis where it stood at 2%.

The previous findings are compatible with the following economic explanation. In immediate fashion, the interest rate reduces consumption by making credit more expensive. Moreover, an increase in rates reduces housing wealth immediately since it makes financing dearer for builders. In 2 years there is a fall in the growth rate of housing prices due to the fall in demand. The fall in the growth rate of house prices puts the brake on new housebuilding even more, on the supply side. The behaviour of housing wealth and housing prices has effects on consumption by reducing it further. However, as the price of housing (and inflation) falls, the central bank begins to reduce interest rates, thus generating a positive effect on consumption. As the periods go by, both the negative effects and the positive ones run out till, finally, the total drop in consumption is 1.33%.

It is important to note that the nature of the reactions to permanent rate changes is due to the integration orders of the variables. Thus, since $r_t$ is $I(1)$, a permanent increase in $r_t$ has permanent effects on $c_t$ and transitory ones on $v_{ct}$. Since both $w_t$ and $p_{vt}$ are $I(2)$, a permanent increase in $r_t$ has permanent effects on $v_{wt}$ as well as on $v_{pvt}$.

VI. Estimation of the Components of Consumption Reaction

In this section, an analysis on the effect of the interest rate on consumer reaction is made by separating its different components. Thus, the effect of the interest rate subdivides into a direct effect brought on by the rise in the cost of credit and an indirect effect resulting from variations in housing wealth and its price. This analysis makes it possible to reach the conclusion as to what the most important factors in the fall in consumption are.

From estimates of $v_c(B)$, $v_w(B)$ and $v_p(B)$, the estimates of $\Gamma_c(B)$, $\Theta_w(B)$, $\Theta_p(B)$ and $\Upsilon_{cp}(B)$ can be obtained (Appendix 2). Table 9 shows these findings. It is important to note that, because $v_{pw} = 0$, then $\Upsilon_{cp}(B) = 0$ too; that is, no response of the central bank to changes in wealth is detected.

As can be seen in Table 9, the effect exerted by the interest rate upon long-term consumption can be separated into the following: (1) a 0.53% drop due to the direct effect, (2) a 0.96% fall due to the role of housing wealth, (3) a 0.48% fall as a result of the unidirectional effect of housing prices and (4) a 0.66% rise due to the feedback effect also generated by housing prices.

It can be observed in more detailed form, that if the interest rate shows a permanent 1% rise, there is an immediate, direct 0.13% drop in consumption. In the following two periods, the direct fall in consumption grows steadily till it reaches 0.43%. This fall represents 80% of the fall generated by the interest rate, in direct form, on consumption and shows how much of the effect is produced in the first 2 years.
from the feedback effect, a positive one on consumption, the growth rate of housing wealth and the growth rate of housing prices. All these findings are in line with most of the previous works. Specifically, a permanent one percentage point increase in rates gives rise to the following: (1) A permanent drop in the growth rate of wealth of 0.15 percentage points. (2) A permanent fall of 0.9 percentage points in the growth rate of housing prices. This finding would partially account for the increase in housing prices in Spain from the mid-1990s. (3) A permanent drop in consumption of 1.33 percentage points. Of these, 0.53 points are a result of the direct effect of rates on consumption (by making credit more expensive), 0.96 points to housing wealth effect, 0.48 points to the unidirectional effect of house prices by reducing collateral and −0.66 points from the feedback effect, a positive one on consumption, which brings with it a reduction in rates decided by the central bank as a response to an easing in prices.

In the long run, the most important component of the consumption final response is that of housing wealth, followed by the direct component and the price component. In the short run, the direct component, as would be expected, dominates the others.

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References


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**Appendix 1: Bands of Confidence for the Response Functions**

Bands of confidence are obtained by means of the bootstrap method (Table A1). By using this method with 100 iterations, the sample distribution of responses is obtained. The confidence bands at 95% are obtained in accordance with the criteria of Efron and Tibshirani (2003).

**Appendix 2: Algebraic Appendix**

Breakdown of effects: proof

From matrices inversion lemma, we have

\[(A - gh')^{-1} = \left(I + A^{-1} \frac{gh'}{1 - \lambda A^{-1}g} \right)A^{-1}\]
### Table A1. Bands of confidence for the Step Response Function (SRF) of \( r \)

<table>
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<tr>
<th>Years</th>
<th>Lower</th>
<th>Higher</th>
<th>Lower</th>
<th>Higher</th>
<th>Lower</th>
<th>Higher</th>
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</table>

Assuming \( A = I \), \( g = v_c(B) \) and \( h' = v_u(B) \), we have

\[
[I - v_c(B)v_c(B)]^{-1} = I + \frac{v_c(B)v_u(B)}{1 - v_c(B)v_c(B)} \tag{A1} \]

where

\[
v_c(B)v_u(B) = \begin{pmatrix} v_u(B) \\ v_c(B) \\ v_c(B)v_u(B) \end{pmatrix} \cdot \begin{pmatrix} v_u(B) & v_u(B)v_u(B) & v_u(B)v_c(B) \\ v_u(B)v_u(B) & v_u(B)v_u(B) & v_u(B)v_c(B) \\ v_c(B)v_u(B) & v_c(B)v_u(B) & v_c(B)v_c(B) \end{pmatrix}
\]

and

\[
v_u(B)v_c(B) = (v_u(B), v_u(B), v_u(B)) \cdot \begin{pmatrix} v_u(B) \\ v_u(B) \\ v_u(B) \end{pmatrix}
\]

Thus

\[
1 - v_u(B)v_u(B) = 1 - [v_u(B)v_u(B) + v_u(B)v_u(B) + v_u(B)v_u(B)]
\]

and

\[
\begin{pmatrix} v_c(B)v_c(B) \\ v_c(B)v_u(B) \\ v_c(B)v_c(B) \end{pmatrix} = \begin{pmatrix} v_u(B)v_u(B) & v_u(B)v_u(B) & v_u(B)v_u(B) \\ v_u(B)v_u(B) & v_u(B)v_u(B) & v_u(B)v_u(B) \\ v_u(B)v_u(B) & v_u(B)v_u(B) & v_u(B)v_u(B) \end{pmatrix} \cdot \begin{pmatrix} v_u(B) \\ v_u(B) \\ v_u(B) \end{pmatrix}
\]
Then

\[
I + \frac{v_z(B)v_n(B)}{1 - v_r(B)v_z(B)} = \left( \begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{array} \right) + \left( \begin{array}{ccc}
v_r(B)v_n(B) & v_r(B)v_p(B) & v_r(B)v_p(B) \\
1 - v_r(B)v_z(B) & 1 - v_r(B)v_z(B) & 1 - v_r(B)v_z(B) \\
v_r(B)v_p(B) & v_r(B)v_p(B) & v_r(B)v_p(B) \\
1 - v_r(B)v_z(B) & 1 - v_r(B)v_z(B) & 1 - v_r(B)v_z(B) \\
\end{array} \right)
\]

\[
= \left( \begin{array}{ccc}
1 + \frac{v_r(B)v_n(B)}{1 - v_r(B)v_z(B)} & \frac{v_r(B)v_n(B)}{1 - v_r(B)v_z(B)} & \frac{v_r(B)v_p(B)}{1 - v_r(B)v_z(B)} \\
\frac{v_r(B)v_n(B)}{1 - v_r(B)v_z(B)} & \frac{1 - v_r(B)v_z(B)}{1 - v_r(B)v_z(B)} & \frac{v_r(B)v_p(B)}{1 - v_r(B)v_z(B)} \\
\frac{v_p(B)v_n(B)}{1 - v_r(B)v_z(B)} & \frac{v_p(B)v_n(B)}{1 - v_r(B)v_z(B)} & \frac{1 - v_r(B)v_z(B)}{1 - v_r(B)v_z(B)} \\
\end{array} \right)
\]

Since

\[
1 + \frac{v_r(B)v_n(B)}{1 - v_r(B)v_z(B)} = \frac{[I - v_r(B)v_z(B)] + v_r(B)v_n(B)}{1 - v_r(B)v_z(B)}
\]

a detailed from (1) becomes

\[
\begin{pmatrix}
[I - v_r(B)v_z(B)] + v_r(B)v_n(B) & v_r(B)v_n(B) & v_r(B)v_p(B) \\
1 - v_r(B)v_z(B) & 1 - v_r(B)v_z(B) & 1 - v_r(B)v_z(B) \\
v_p(B)v_n(B) & v_p(B)v_n(B) & v_r(B)v_p(B) \\
1 - v_r(B)v_z(B) & 1 - v_r(B)v_z(B) & 1 - v_r(B)v_z(B) \\
v_r(B)v_n(B) & v_r(B)v_n(B) & 1 - v_r(B)v_z(B) \\
1 - v_r(B)v_z(B) & 1 - v_r(B)v_z(B) & 1 - v_r(B)v_z(B) \\
\end{pmatrix}
\]

\[
\Psi_r(B) = \left( \begin{array}{c}
[I - v_r(B)v_z(B)] + v_r(B)v_n(B) \\
1 - v_r(B)v_z(B) \\
v_p(B)v_n(B) \\
1 - v_r(B)v_z(B) \\
v_r(B)v_n(B) \\
1 - v_r(B)v_z(B) \\
\end{array} \right) \cdot \pi_r(E)
\]

\[
\Psi_{v_n}(B) = \frac{[I - v_r(B)v_z(B)] + v_n(B)v_r(B)v_n(B)}{1 - v_r(B)v_z(B)} \pi_r(B)^{-1} + \left[ \frac{v_r(B)v_n(B)v_n(B)}{1 - v_r(B)v_z(B)} + \frac{v_r(B)v_p(B)v_p(B)}{1 - v_r(B)v_z(B)} \right] \pi_r(B)^{-1}
\]

If \( v_r(B) = 0 \), then

\[
\Psi_r(B) = \left[ v_r(B) + \frac{v_r(B)v_n(B)v_n(B)}{1 - v_r(B)v_z(B)} + \frac{v_r(B)v_p(B)v_p(B)}{1 - v_r(B)v_z(B)} \right] \pi_r(B)^{-1}
\]

where

\[
1 - v_r(B)v_z(B) = 1 - [v_n(B)v_n(B) + v_p(B)v_p(B)]
\]
**Consumption and housing wealth**

Since \( v_c(B) = 0 \), Equation A4 becomes

\[
\Psi_{rc}(B) = v_c(B)\pi_c(B)^{-1} + \frac{v_c(B)v_w(B)v_p(B)}{1 - [v_w(B)v_p(B) + v_p(B)v_p(B)]}\pi_c(B)^{-1} + \frac{v_c(B)v_w(B)v_p(B)}{1 - [v_w(B)v_p(B) + v_p(B)v_p(B)]}\pi_c(B)^{-1}
\]

\[(A5)\]

Equation A5 shows that \( \Psi_{rc}(B) \) can be broken down into two components: (1) an unidirectional response from interest rate to consumption, represented by \( v_c(B) \), and (2) the sum of two feedback effects:

(i) the feedback effect due to the residential wealth is

\[
v_c(B)v_w(B)v_p(B)\pi_c(B)^{-1} \quad \text{where} \quad \pi_c(B)^{-1} \quad \text{is the unidirectional indirect effect of interest rates on consumption, throughout housing, because \( \Theta_u(B) \) is the combination of two effects: (1) the effect of interest rate on residential wealth, represented by \( v_w(B) \) and (2) the effect of housing prices on consumption, represented by \( \pi_c(B). \)

(ii) the feedback effect due to the housing price is

\[
\frac{v_c(B)v_p(B)v_p(B)}{1 - [v_w(B)v_p(B) + v_p(B)v_p(B)]}\pi_c(B)^{-1}
\]

If we represent the residential wealth feedback effect by \( \psi_{rw}(B) \) and the housing price feedback effect by \( \psi_{cp}(B) \), then the reaction of consumption to the interest rate can be written as follows:

\[
\psi_{rc}(B) = v_c(B) + \psi_{rw}(B) + \psi_{cp}(B) \quad (A6)
\]

\[
\begin{bmatrix}
\pi_{11}(B) & \pi_{12}(B) & \pi_{13}(B) \\
\pi_{21}(B) & \pi_{22}(B) & \pi_{23}(B) \\
\pi_{31}(B) & \pi_{32}(B) & \pi_{33}(B) \\
\end{bmatrix}
\begin{bmatrix}
-\pi_{11}(B)v_c(B) - \pi_{12}(B)v_w(B) - \pi_{13}(B)v_p(B) \\
-\pi_{21}(B)v_c(B) - \pi_{22}(B)v_w(B) - \pi_{23}(B)v_p(B) \\
-\pi_{31}(B)v_c(B) - \pi_{32}(B)v_w(B) - \pi_{33}(B)v_p(B)
\end{bmatrix}
\begin{bmatrix}
\alpha_c \quad \alpha_w \quad \alpha_p
\end{bmatrix}
\]

\[(A10)\]

Moreover, it was pointed out in Section II that the direct effect of the interest rate on consumption can be obtained from the structural model and takes the form

\[
\Gamma_i(B) = \frac{v_c(B)}{\pi_{11}(B)}v_w(B) + \frac{\pi_{13}(B)}{\pi_{11}(B)}v_p(B) \quad (A7)
\]

Then,

\[
v_c(B) = \Gamma_i(B) + \Theta_u(B) + \Theta_p(B) \quad (A8)
\]

where

\[
\Theta_u(B) = \frac{\pi_{13}(B)}{\pi_{11}(B)}v_p(B)
\]

is the unidirectional indirect effect of interest rates on consumption, throughout housing, because \( \Theta_u(B) \) is the combination of two effects: (1) the effect of interest rate on residential wealth, represented by \( v_w(B) \) and (2) the effect of housing prices on consumption, represented by \( \pi_c(B). \)

Also

\[
\Theta_p(B) = \frac{\pi_{13}(B)}{\pi_{11}(B)}v_p(B)
\]

Finally, Equation A9 is obtained by substituting Equation A8 in Equation A6

\[
\psi_{rc}(B) = [\Gamma_i(B) + \Theta_u(B) + \Theta_p(B) + \psi_{rw}(B) + \psi_{cp}(B)]\pi_c(B)^{-1} \quad (A9)
\]

**Identification of the polynomials in the consumption response function**

The theoretical model presented in Section II can be written in its most detailed form as

\[
VY_t = VD + V(I_k + \Pi + \Gamma_1)Y_{t-1} + V(\Gamma_2 - \Gamma_1)Y_{t-2} - VY_{t-3} + \epsilon_t
\]

From an estimated VEC model and using the following equation, we have

\[
(\text{A11})
\]
The orthogonalized VAR model in Equation A10 can be estimated as

\[
\begin{pmatrix}
(1 - 1.47B)\nu - 2.31B^2\nu - 0.19BV \\
(1 - 0.352B - 0.226B^2)\nu - 0.01BV \\
0.00 - 5.84B - 1 - 0.24B \\
0.00 0.00 - 0.38\nu - 1 - B
\end{pmatrix}
\begin{pmatrix}
on \\ \ \ \epsilon c_t \\
\epsilon c_w \\
\epsilon cpv \\
\epsilon cr
\end{pmatrix}
\begin{pmatrix}c_t \\
\nu w_t \nu \\
\nu pv_t \\
v_t \end{pmatrix}
\]  

(A12)

that is, the estimated version of the theoretical model proposed in Equation A3. From Equation A12, we get the following estimates:

\[
- \pi_{11}(B)\nu_{11}(B) - \pi_{12}(B)\nu_{12}(B) - \pi_{13}(B)\nu_{13}(B) = a(B)
\]

\[
- \pi_{21}(B)\nu_{21}(B) - \pi_{22}(B)\nu_{22}(B) - \pi_{23}(B)\nu_{23}(B) = b(B)
\]

\[
- \pi_{31}(B)\nu_{31}(B) - \pi_{32}(B)\nu_{32}(B) - \pi_{33}(B)\nu_{33}(B) = b(B)
\]

(A13)

and \( \pi(B) \) polynomials in Equation A10 can be obtained directly.

The system (A13) can be solved using a computer program of numerical and symbolic calculus, yielding estimates of \( \nu_{11}(B), \nu_{12}(B) \) and \( \nu_{13}(B) \):

\[
\nu_{11}(B) = \left\{ \frac{-0.126 - 0.087B + 0.10B^2}{1 - 1.06B + 0.08B^2 + 0.14B^3 - 0.25B^4} \right\}
\]

(A14)

\[
\nu_{12}(B) = \left\{ \frac{-0.023 - 0.024B - 0.02B^2 + 0.007B^3}{1 - 0.59B - 0.19B^2 + 0.054B^3} \right\}
\]

(A15)

\[
\nu_{13}(B) = \left\{ \frac{1.16B - 0.28B^2 - 0.24B^3}{1 - 0.59B - 0.19B^2 + 0.054B^3} \right\}
\]

(A16)

### Appendix 3: Data

The data are presented in Table A2.

<table>
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<tr>
<th>Years</th>
<th>Consumption</th>
<th>Housing wealth</th>
<th>Price of housing</th>
<th>Interest rate</th>
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